Matched Filter CNR, Diversity and Signal Detectivity for Deterministic and Random Coherent Ladar Signals

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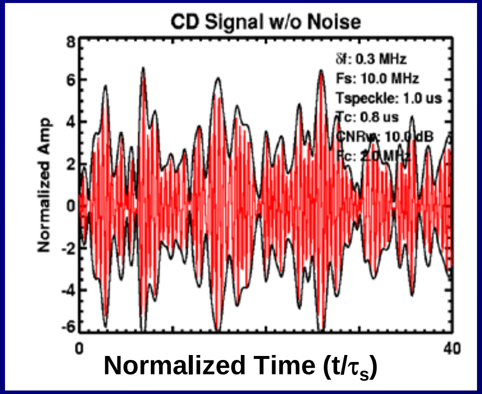
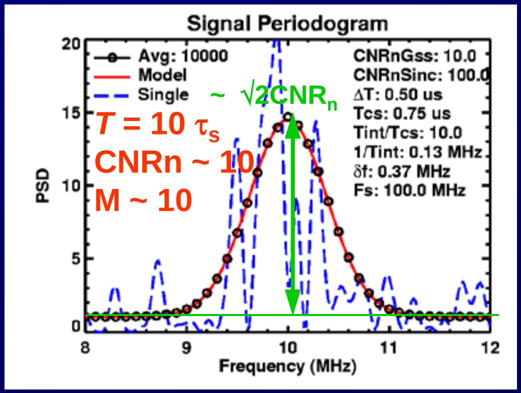
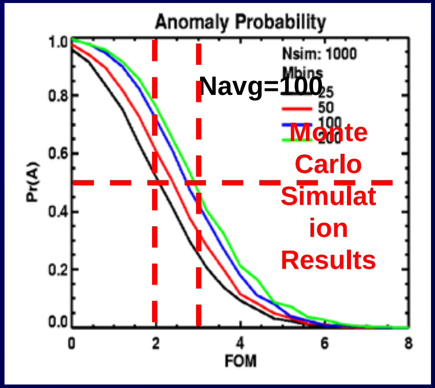
Introduction

- We develop theoretical expressions for coherent ladar signal:
  - Wideband and matched filter CNR
  - Speckle coherence time
  - Intrinsic diversity (a measure of speckle noise mitigation, *M* = Speckle SNR)
  - Spectrum models valid for arbitrary integration times
    - long or short relative to the coherence time
  - Relationship between spectrum height and matched filter CNR (CNRn) is developed
  - Relationship to signal detectivity

- Theoretical predictions are supported by Monte-Carlo simulation experiment results





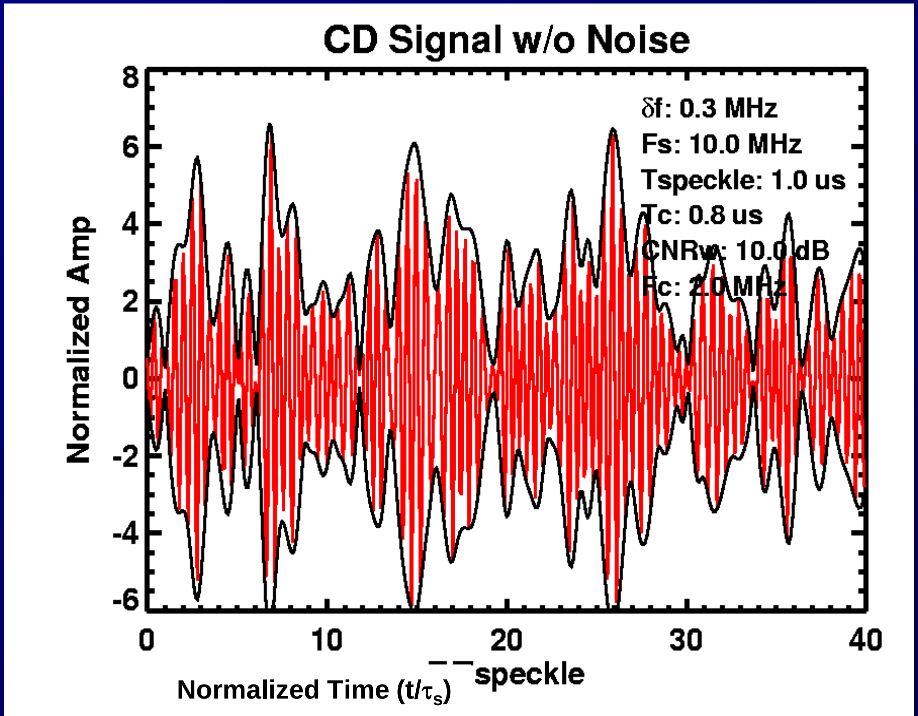


Coherent Ladar CNR

$$CNR_w = \frac{\eta_r \Pr}{h\nu B} = \frac{\eta_r \Pr T}{h\nu BT} = \frac{\eta_r E_r}{h\nu BT} = \frac{m_r}{BT}$$

- ***For signal integration over times short compared to the speckle coherence time***

$$B = \frac{1}{T} \quad and$$
$$CNR_n = m_r$$



Coherent Ladar CNR

- ***For arbitrary integration times, T***

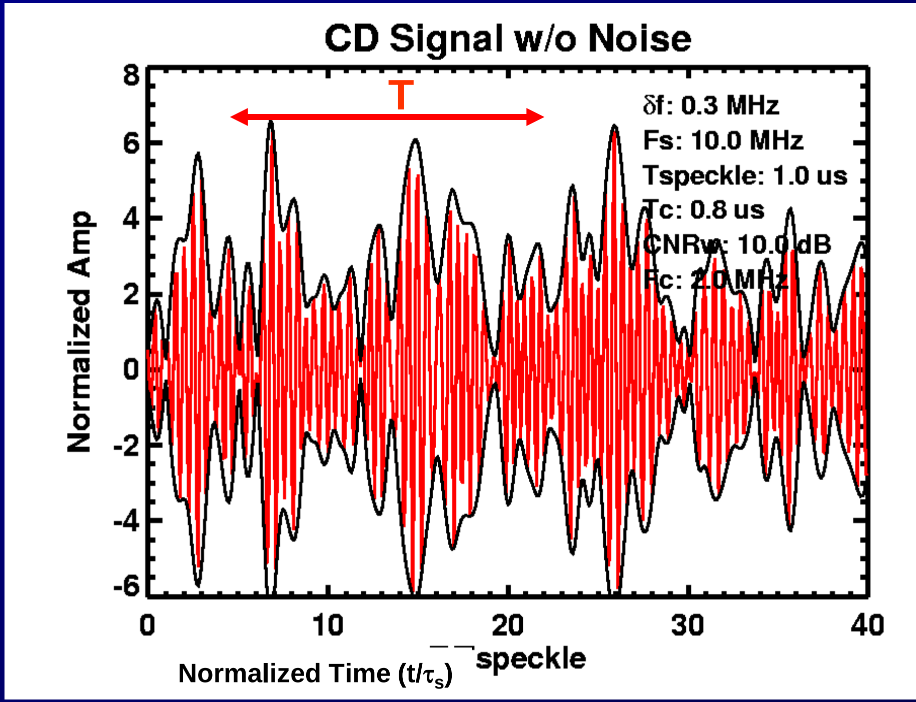
$$CNR_w = \frac{\eta_r \Pr}{h\nu B} = \frac{m_r}{BT}$$

- ***Optimal matched filter Bandwidth related to characteristic speckle time***

$$B = \frac{1}{\tau_s} \quad and$$

$$CNR_n = \frac{\tau_s}{T} m_r = \frac{m_r}{M}$$

$$for\ T > \tau_s = m_{rs}$$



What are detailed descriptions of τ_s and M?

Long Duration ($T >> \tau_s$) Signal Diversity and Speckle Coherence Time

- Diversity is defined as the speckle limited signal SNR or reciprocal normalized power variance

$$M \equiv \frac{E[P_s]^2}{\mathrm{var}[P_s]} = SNR_{speckle}$$

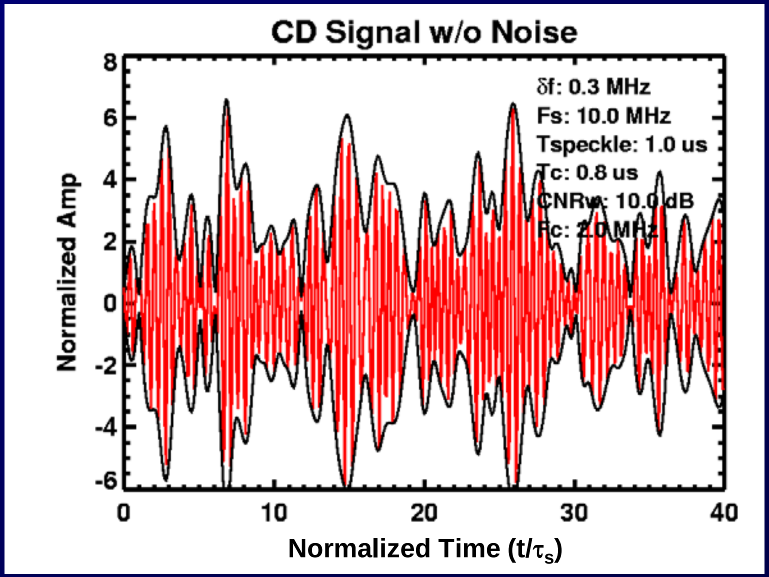
- There are many definitions for coherence time.
  - For very long integration times, *T*, the definition that leads to the diversity, *M*, converging to $M = T/\tau_s$ is what we call the "speckle coherence time"

$$M \approx T/\tau_S \ \text{ for } \ T >> \tau_S$$

- Goodman shows, Ch 6 Statistical Optics, that this coherence time is given by

$$\tau_S = \int_{-\infty}^{\infty} |\Gamma(\tau)|^2 d\tau / \Gamma(0)^2 = \int_{-\infty}^{\infty} |\gamma(\tau)|^2 d\tau$$

  - This is a measure of the width of the normalized signal autocorrelation function
  - Parseval's Theorem leads to an expression in terms of the signal spectrum (not shown)



- For a Gaussian autocorrelation function

$$\tau_S = \sqrt{\pi/2}\,\tau_C = 1.25\tau_C$$

  - Where τ_c is the exp[-1] coherence time

Matched Filter CNR and Integrated Speckle Diversity for Arbitrary Duration Signals

- For arbitrary duration signals, the matched filter CNR can be generalized
  - Coherent photoelectrons contribute to CNRn
  - Incoherent photoelectrons contribute to diversity
  - Total #electrons = CNRn and diversity product

$$CNR_n = \frac{m_r}{M}$$

$$m_r = M\, CNR_n$$

For arbitrary duration signals, a general expression for the diversity of the temporally integrated coherent signal power is given by [Goodman Ch. 6]

$$M^{-1} = \frac{1}{T}\int_{-\infty}^{\infty} \Lambda(t/T)\left|\gamma(\tau)\right|^2 d\tau$$

$\Lambda(t)$ is the triangle function
$\gamma(t)$ is the normalized autocorrelation function

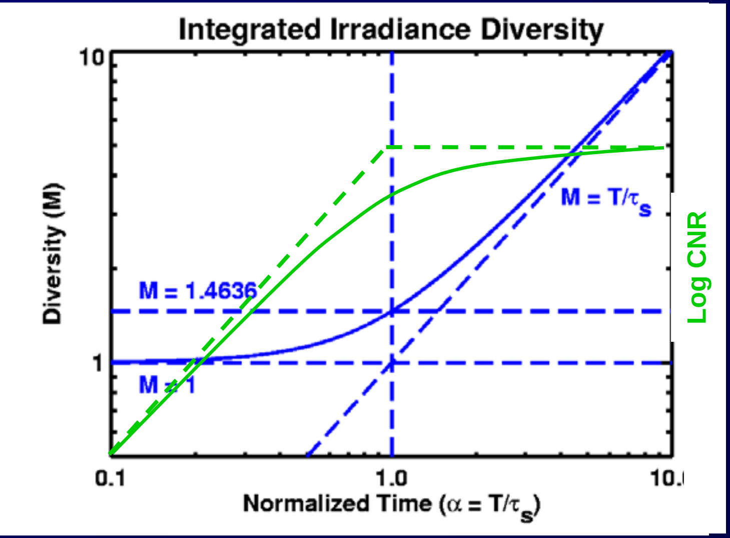
- For long integration times, the triangle function can be approximated as 1 and the integral converges to the speckle coherence time (see previous chart), leading to

$$M \to T/\tau_s; \quad \text{for } T >> \tau_s$$

- For a Gaussian signal spectrum, this becomes

$$M^{-1} = \frac{\operatorname{erf}(\sqrt{\pi}\alpha)}{\alpha} - \frac{1}{\pi\alpha^2}\left(1-\exp(-\pi\alpha^2)\right)$$

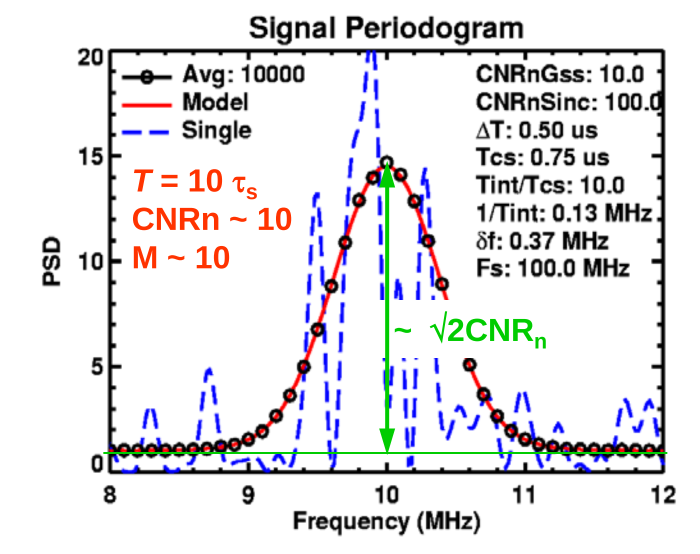
  - where $\alpha = T/\tau_s$
  - For small α, $M \Rightarrow 1$
  - For Large α, $M \Rightarrow \alpha = T/\tau_s$
  - when $\alpha = 1$, $M = 1.4636$



Spectral Questions to Consider

- What is the spectral model as a function of the integration time relative to the coherence time.

- How is the matched filter CNR related to the ratio of the spectral peak to the noise floor?
  - Coherent CW signal: CNRn
  - Coherent Gaussian pulse: √2 CNRn
  - Long duration random signal with Gaussian autocorrelation/spectrum: √2 CNRn



Fully Coherent CW Signal: Short Duration ($T << \tau_s$) Signal Spectral Model

- FFT is a matched filter for a rectangular windowed sine wave
- Peak of FFT output should be E_s/N_{oe} = CNR_n
  - Because it is the matched filter

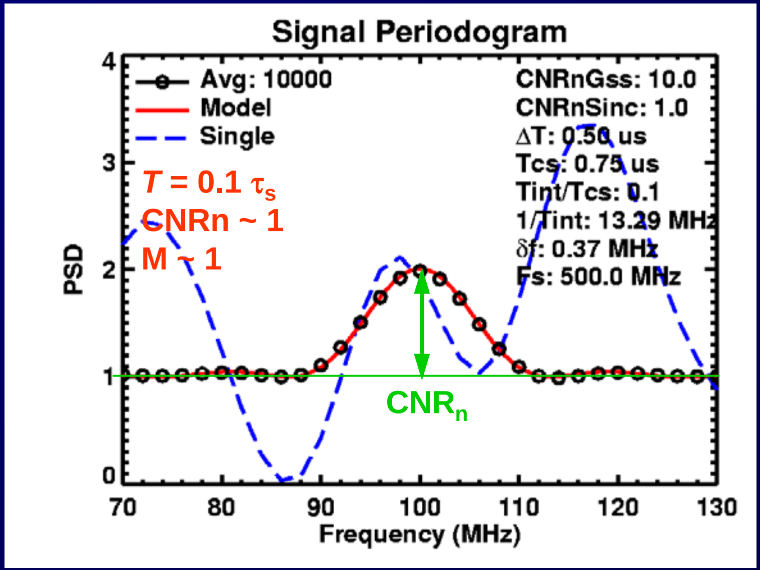
- Signal Spectrum is a sinc function

$$S_s(f) = CNR_n \,\mathrm{sinc}^2((f - f_c)T) \qquad \mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$$

- So with noise the final model is

$$S_{sn}(f) = (No/2)(1 + CNR_n)\,\mathrm{sinc}^2((f - f_c)T)$$

- The peak spectral height above the unit normalized NSD is precisely equal to the matched filter CNR, CNRn



Arbitrary Duration Spectral Model

- An arbitrary duration signal is the product of a rectangular window with an infinite duration signal.

$$s_T(t) = w(t)s(t)$$

- For single spectral realizations, a multiply in time domain implies
  - Fourier transforms convolve $S_T(f) = W(f) * S(f)$
  - and spectrum, $S_{sT}(f) = |S_T(f)|^2$

- For ensemble average spectra, a multiply in time domain implies
  - Power spectra convolve $S_{sT}(f)=S_w(f)*S_s(f)$
    - See Goodman's Gaussian moment theorem

$$S_{sn}(f) = (No/2)\left(1 + \frac{S_{sinc}(f) * S_s(f)}{\int S_{sinc}(f)df}\right)$$

    - Signal Spectrum convolved with a unit area sinc^2.
    - Unit area sinc^2 normalization ensures noise PSD remains constant with dwell time variations
    - For a Gaussian signal spectrum (see backup charts)

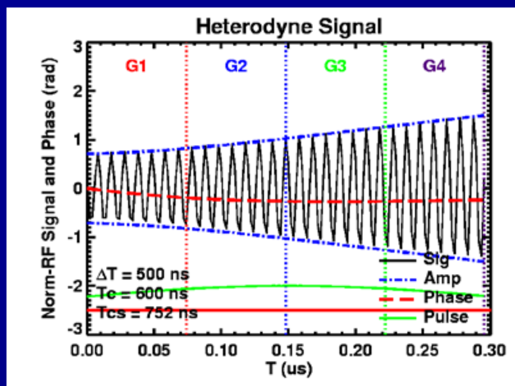
$$S_s(f) = \sqrt{2}CNR_n \exp\left(\frac{(f - fc)^2}{2\delta f^2}\right)$$

Monte Carlo Simulation Experimental Results for Arbitrary Integration Times Agree with Theory

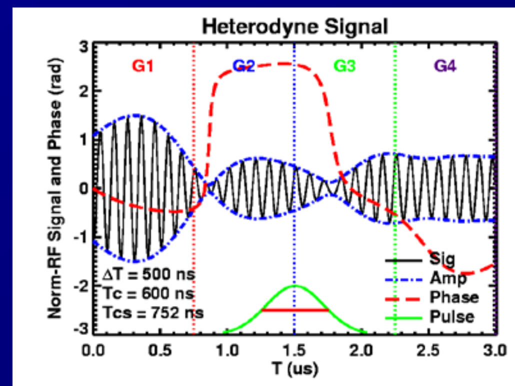
- 10,000 Spectral Avg Monte Carlo Simulation
- Results agree with theory for arbitrary integration time

Time Domain

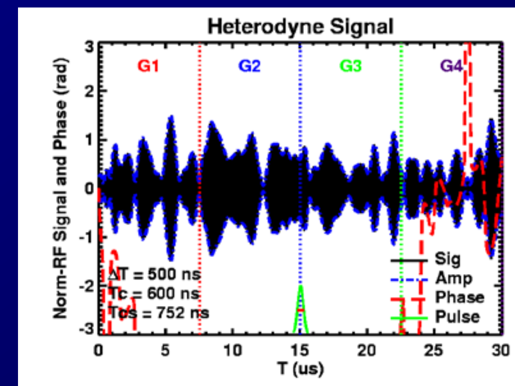
$T/\tau_s = 0.1$



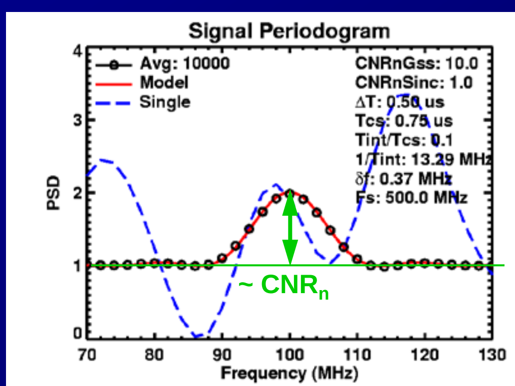
$T/\tau_s = 1.0$

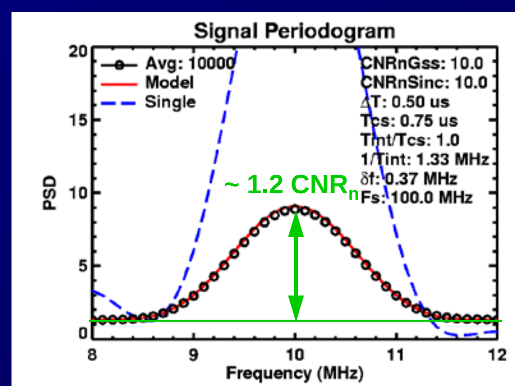


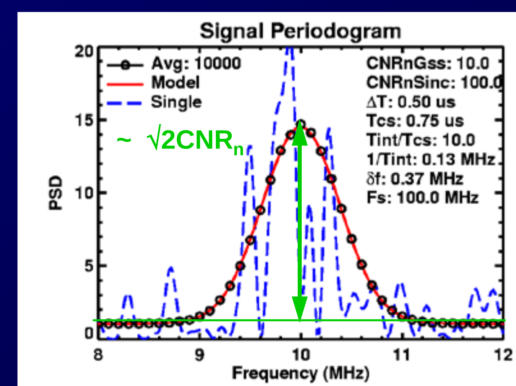
$T/\tau_s = 10$



Spectrum vs. Model

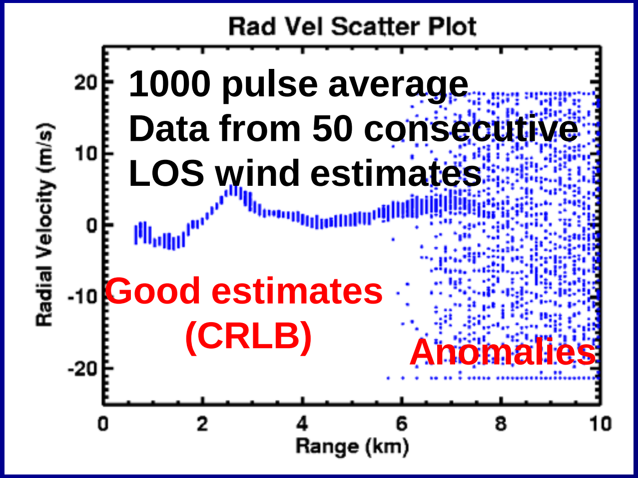






Signal Detectivity

- Detectivity relates to sensor range performance

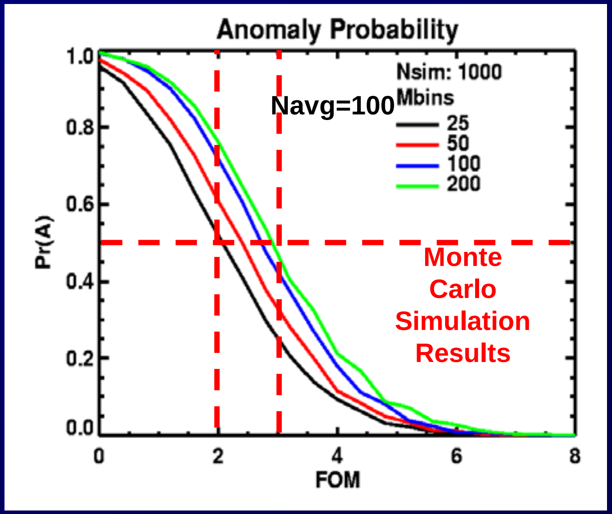


- Detectivity is defined as the ratio of the peak signal spectral height above the noise, to the rms fluctuations in the noise. Assume unit normalized NSD.
  - Spectral peak above noise is *k* CNRn
    - where *k* is a constant close to 1
  - Noise rms is $1/\sqrt{N}$
    - gamma distributed

- Consequently, the detectivity or Figure of Merit is given by

$$FOM = k\sqrt{N}\,CNR_n$$

- This FOM can be utilized to characterize the anomaly probability for a peak-detecting estimator algorithm



- So for 2 < FOM < 3, PrA ~ 50% depending on number of noise bins

Summary

- Matched Filter CNR

$$CNR_n = \frac{m_r}{M}$$

Coherent photons build up CNRn
Incoherent photons build up diversity

- Diversity
  - Fully coherent CW signal

$$M = 1$$

  - Partially coherent CW signal

$$M^{-1} = \frac{1}{T}\int_{-\infty}^{\infty} \Lambda(t/T)\left|\gamma(\tau)\right|^2 d\tau$$

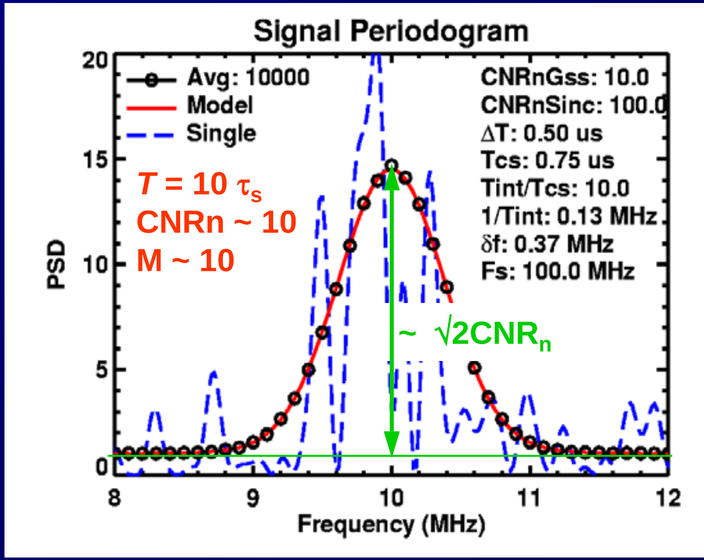
- Speckle Coherence Time

$$\tau_s = \int_{-\infty}^{\infty} \left|\gamma(\tau)\right|^2 d\tau$$

- General Spectrum Model

$$S_{sn}(f) = (No/2)\left(1 + \frac{S_{sinc}(f) * S_s(f)}{\int S_{sinc}(f)df}\right)$$

- Peak above the normalized noise floor
  - Sp = 1.0 CNRn for fully coherent signals
  - Sp ~ 1.2 CNRn for *T*/ts = 1.0
  - Sp = 1.414 CNRn for infinite duration incoherent signals



- Signal Detectivity

$$FOM = Sp\sqrt{N} \sim CNR_n\sqrt{N}$$

- Coherent Receiver Sensitivity / Pulse
  - ~ (1 coherent photon/η_r) / $\sqrt{N}$