Ensemble copula coupling (ECC): Application to the ECMWF ensemble

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Introduction

- Forecast ensembles typically require statistical postprocessing to address biases and dispersion errors.
- Widespread state-of-the-art univariate ensemble postprocessing methods are
  - Bayesian model averaging (BMA) [6] and
  - Ensemble model output statistics (EMOS) [2].
- These methods only apply to a *single* weather quantity at a *single* location for a *single* look-ahead time.
- However, physical consistency of multivariate dependencies across space, time and variables is required in numerous applications, such as air traffic management.
- Recent multivariate ensemble postprocessing methods include among others
  - Bivariate postprocessing of (u, v)- wind vectors [5] [7] and a
  - Gaussian copula approach [4].

  Our proposal: **ensemble copula coupling (ECC)**

  discrete copula-based and non-parametric approach, which retains the spatial, temporal and inter-variable rank dependence patterns of the raw ensemble

The ECC approach

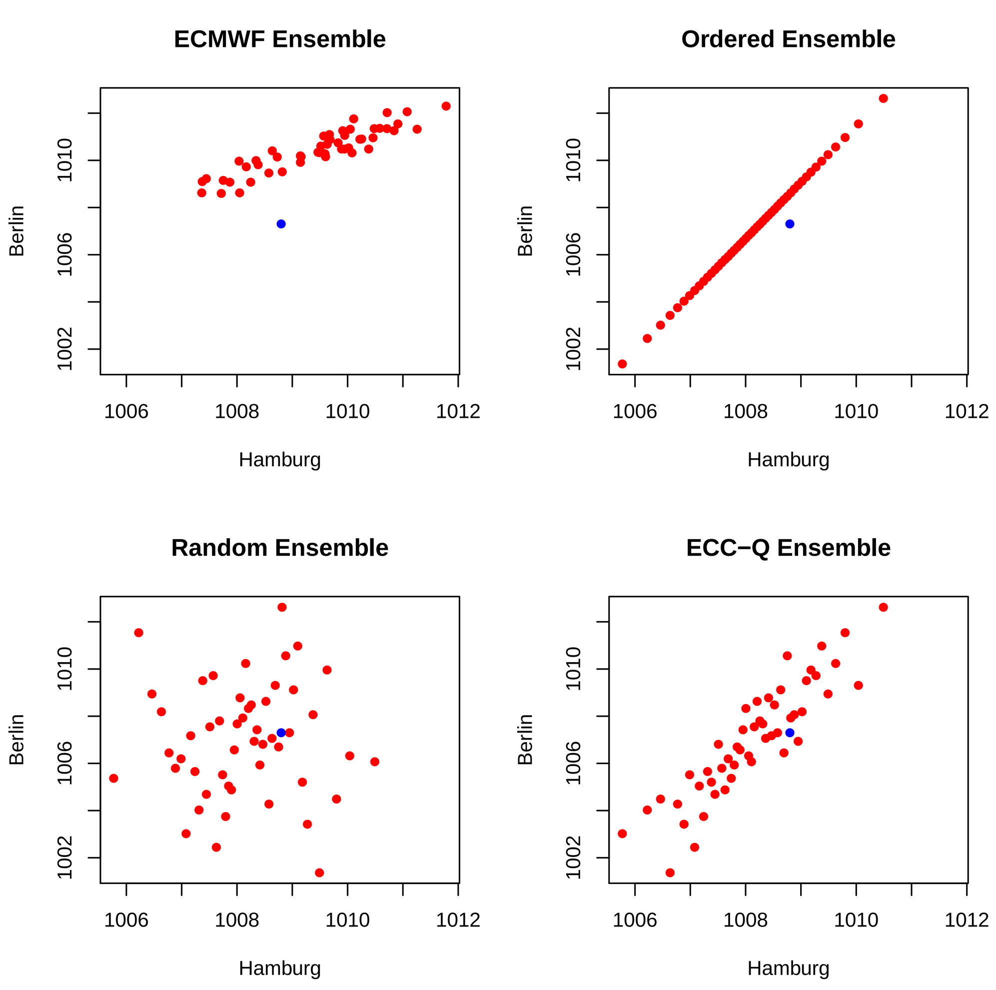

Figure 1: *48 hour pressure forecasts (in hPa) at Berlin and Hamburg valid 0000 UTC on 18 September 2010. The verifying observation is indicated by the blue dot.*

1. **Raw ensemble:** For each variable $i \in \{1, \ldots, I\}$, location $j \in \{1, \ldots, J\}$ and prediction horizon $k \in \{1, \ldots, K\}$, find the raw ensemble forecast $x_1^\ell, \ldots, x_M^\ell$, where $\ell := (i, j, k) \in \{1, \ldots, L\}$, and L is the number of possible combinations of i, j and k.
   For each fixed ℓ, determine the corresponding order statistics $x_{(1)}^\ell \leq \ldots \leq x_{(M)}^\ell$, with ties resolved at random, by computing the permutation σ_ℓ of $1, \ldots, M$ given by
   $\sigma_\ell(m) := \text{rank}\,(x_m^\ell)$ for all $m \in \{1, \ldots, M\}$.
2. **Univariate postprocessing:** Use univariate postprocessing methods to obtain calibrated and sharp predictive distributions F_{X^ℓ} for each variable, location and look-ahead time individually.
3. **Quantization:** For each ℓ, generate M samples $\tilde{x}_1^\ell, \ldots, \tilde{x}_M^\ell$ from F_{X^ℓ}, which can be done in various ways:
   - **ECC-R**: Take simple random samples.
   - **ECC-P**: Take the quantiles of F_{X^ℓ} that correspond to the Pearson residuals of the ensemble values [5].
   - **ECC-Q**: Take the equidistant $(\frac{m-0.5}{M})_{m=1,\ldots,M}$-quantiles of F_{X^ℓ} (idea from e.g. [1]) $\longrightarrow$ our preferred variant.
4. **Reordering/ECC:** For each ℓ, the ECC ensemble is given by $\hat{x}_1 := \tilde{x}^\ell_{(\sigma_\ell(1))}, \ldots, \hat{x}_M := \tilde{x}^\ell_{(\sigma_\ell(M))}$.

By applying the empirical raw ensemble copula to the samples from step 3, it inherits the multivariate rank dependence structure from the raw ensemble.

Multivariate verification methods

Goal of probabilistic forecasting: maximize sharpness subject to calibration.

- Multivariate rank histogram (MRH) [3] to check calibration:
  - flat MRH indicates calibrated ensemble,
  - (inverse) U-shaped MRH indicates underdispersive (overdispersive) ensemble,
  - skewed MRH indicates biased ensemble.
- Energy score (ES) [3] for discrete distributions as an overall performance measure (the lower the better):

$$\text{ES}(P_{\text{ens}}, \mathbf{x}) = \frac{1}{M}\sum_{m=1}^{M} ||\mathbf{x}_m - \mathbf{x}|| - \frac{1}{2M^2}\sum_{n=1}^{M}\sum_{m=1}^{M} ||\mathbf{x}_n - \mathbf{x}_m||,$$

where P_{ens} is the predictive distribution, M the ensemble size, $\mathbf{x}_1, \ldots, \mathbf{x}_M \in \mathbb{R}^L$ the ensemble forecasts, $\mathbf{x} \in \mathbb{R}^L$ the observation vector and $||.||$ the Euclidean norm.

Case study: The ECMWF ensemble over Germany

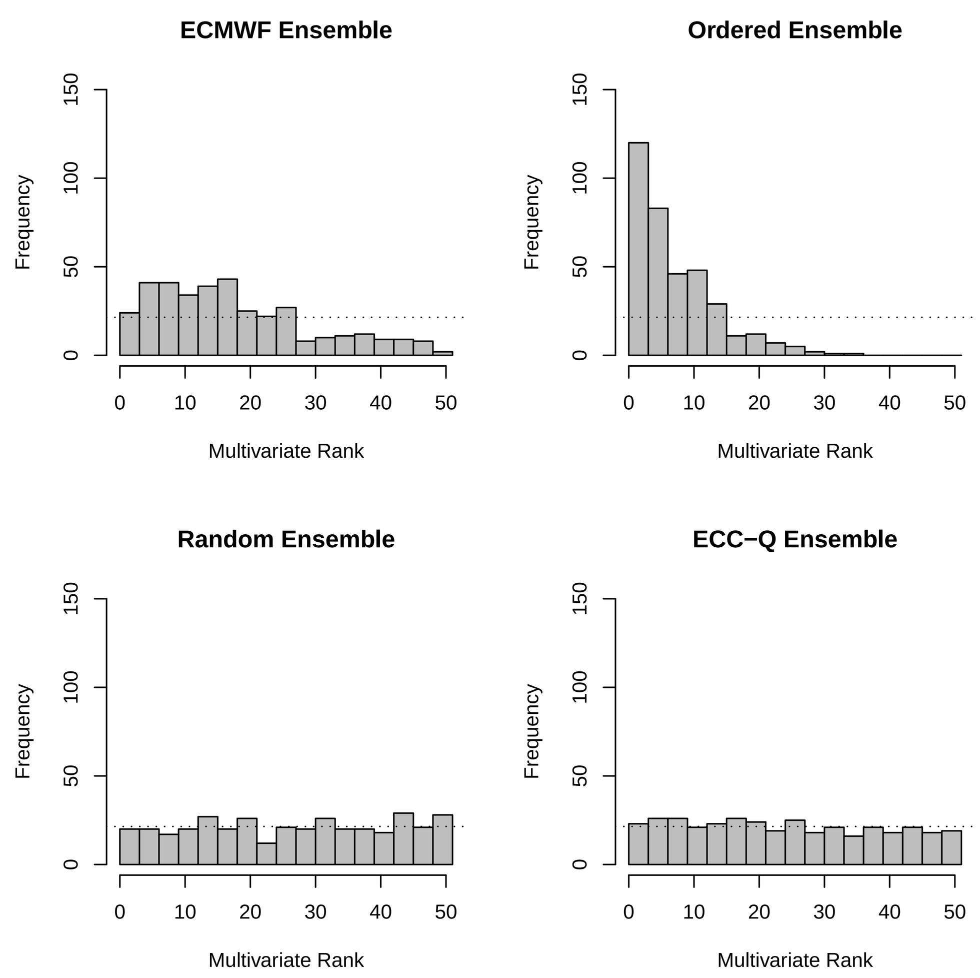

Figure 2: *MRHs for 24 hour u-wind vector forecasts over an area in Upper Bavaria for 1 May 2010 to 30 April 2011*

Table 1: *Different quantization approaches and ECC: Average ES (in m/s) for 48 hour (u, v)-wind vector forecasts for 1 May 2010 to 30 April 2011.*

Ensemble	Berlin	Frankfurt	Hamburg
ECMWF	1.21	1.66	1.30
ECC-R	1.10	1.44	1.18
ECC-P	1.10	1.43	**1.17**
ECC-Q	**1.09**	**1.42**	**1.17**

- European Centre for Medium-Range Weather Forecasts (ECMWF) ensemble with 50 exchangeable members
- Forecasts are available on a grid consisting of 1221 points over Germany and surrounding areas. Use the mean of the next day's 0 h ECMWF model nowcasts as the ground truth.
- Employ bilinearly interpolated forecasts and real observations for the locations Berlin, Hamburg and Frankfurt.
- Univariate postprocessing for each weather variable, location and prediction horizon is done either by BMA or EMOS, using a training period of 30 days.
- Use ECC-Q and sample the equidistant $(\frac{m-0.5}{50})_{m=1,\ldots,50}$-quantiles from each univariate predictive distribution.
- Create and compare different postprocessed ensembles:
  - Ordered quantiles (**OQ**) ensemble: $(\frac{m-0.5}{50})_{m=1,\ldots,50}$-quantiles in **increasing** order,
  - Random quantiles (**RQ**) ensemble: $(\frac{m-0.5}{50})_{m=1,\ldots,50}$-quantiles **randomly** ordered,
  - **ECC-Q** ensemble: $(\frac{m-0.5}{50})_{m=1,\ldots,50}$-quantiles ordered **with respect to the ECMWF raw ensemble ranks**.

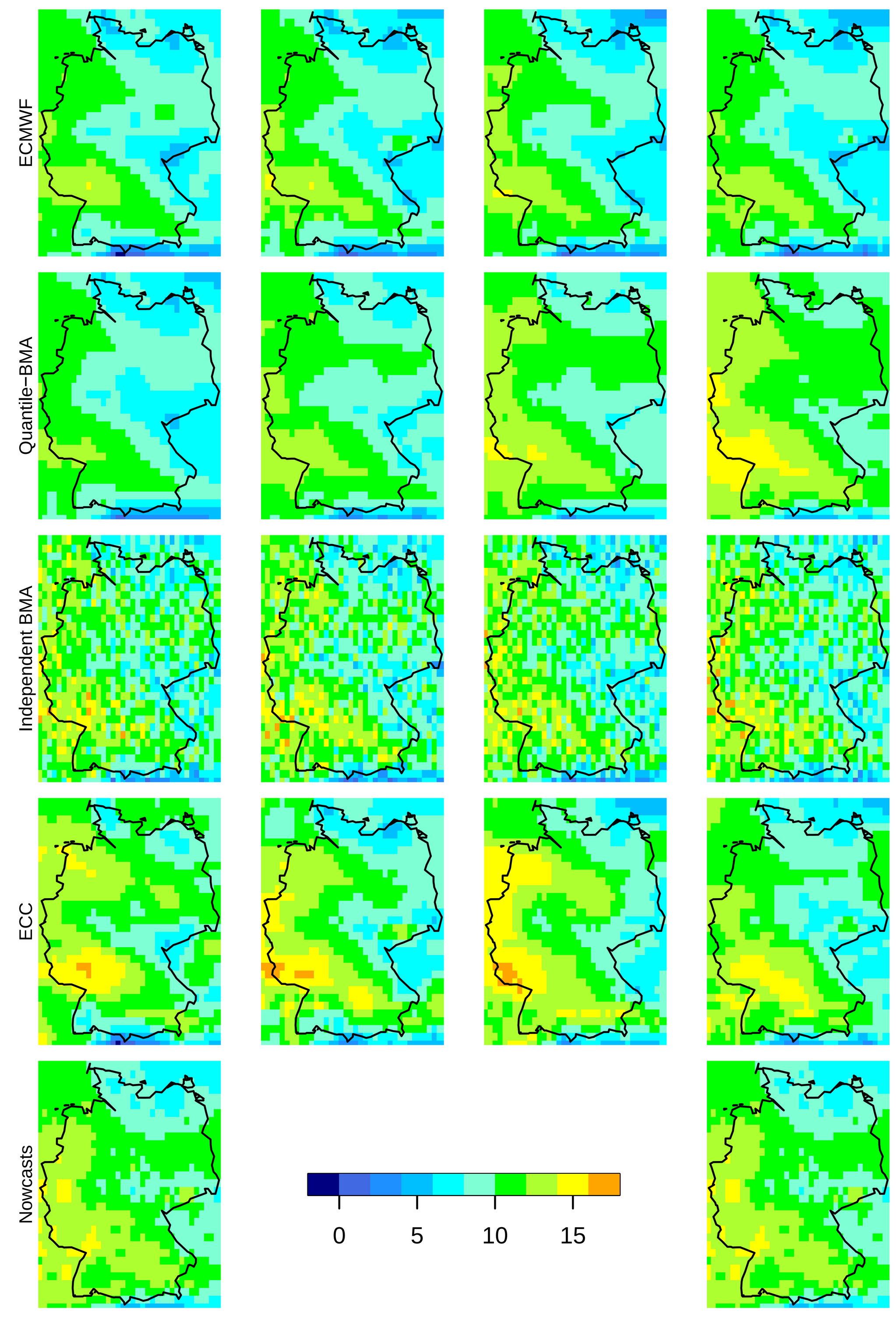

Figure 3: *24 hour temperature forecasts (in °C) over Germany valid 0000 UTC on 25 April 2011. First row: selected ECMWF ensemble members; second row: the 20th, 40th, 60th, and 80th quantiles from the BMA forecasts; third row: random independent realizations from the BMA forecasts; fourth row: ECC forecasts corresponding to the members in the first row; fifth row: the ensemble mean nowcasts.*

Table 2: *Average ES for 24 hour forecasts over contiguous test areas for 1 May 2010 to 30 April 2011.*

		Area I: Baltic Sea	Area II: North Rh.-Westph.	Area III: Upper Bavaria	Area IV: Rhine-Neckar
Temp. (°C)	ECMWF	5.01	5.79	14.22	5.19
	OQ	4.48	5.08	10.63	**4.30**
	RQ	4.50	5.14	10.94	4.47
	ECC-Q	**4.39**	**5.00**	**10.56**	**4.30**
Pressure (hPa)	ECMWF	3.44	3.38	4.66	2.39
	OQ	**2.84**	**3.09**	3.98	**2.07**
	RQ	3.17	3.46	4.22	2.30
	ECC-Q	2.88	3.13	**3.96**	2.13
u-wind (m/s)	ECMWF	4.36	3.75	4.53	2.43
	OQ	4.36	3.78	4.51	2.45
	RQ	4.36	3.72	4.40	2.44
	ECC-Q	**4.25**	**3.66**	**4.33**	**2.40**

Summary of ECC

- Postprocessing tool retaining the spatial, temporal and inter-variable rank dependencies of the raw ensemble
- Connected to discrete/empirical copulas
- Simple and clear, yet powerful and well performing
- Easy to implement given `R` packages for univariate postprocessing, such as `ensembleBMA` or `ensembleMOS`

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